

Appendix A Techincal Details

If X_1, X_2, \dots, X_n are iid $N(\mu, \sigma^2)$ then \bar{X} is $N(\mu, \sigma^2/n)$ and the margin of error for \bar{X} is approximately $1.96\sigma^2/n$ when n is large.

For a normal distribution we expect 95% of the observations to be between $\mu - 1.96\sigma$ and $\mu + 1.96\sigma$. With data, these bounds are estimated by $\bar{X} - 1.96s$ and $\bar{X} + 1.96s$, respectively.

Under the normal assumption, \bar{X} and s are independent, so $\text{var}(\bar{X} \pm 1.96s) = \text{var}(\bar{X}) + 1.96^2\text{var}(s)$. Also, under the normal assumption, $(n-1)s^2/\sigma^2$ is distributed as χ_{n-1}^2 . Therefore, $\text{var}(s)$ is approximately $\sigma^2/(2(n-1))$ and $\text{var}(\bar{X} \pm 1.96s)$ is approximately $3\sigma^2/(n-1)$, and the margins of error for $\bar{X} - 1.96s$ and $\bar{X} + 1.96s$ are both $\sigma\sqrt{3/(n-1)}$. For a 50% coefficient of variation, $\sigma = 0.5\mu$ and the margin of error for the estimates is $\mu\sqrt{3/(n-1)}$.

For $n = 500$ and $\mu = 5$, the margin of error is 0.39; for $n = 500$ and $\mu = 0.5$, the margin of error is 0.039. These results have been verified by simulation.